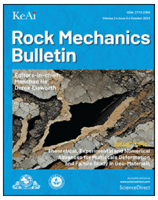
Rock Mechanics Bulletin 2 (2023) 100080

Contents lists available at ScienceDirect

Rock Mechanics Bulletin

journal homepage: www.keaipublishing.com/en/journals/rock-mechanics-bulletin



More is different: On the emergence of collective phenomena in fractured rocks

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ARTICLE INFO

Keywords:
Fracture network
Emergence
Upscaling
Complexity
Hierarchy

ABSTRACT

Fractures widely exist in crustal rocks and form complex networks dominating the bulk behaviour of geological media. Thus, understanding how fracture networks affect subsurface processes/phenomena is highly relevant to many rock engineering applications. However, the large-scale behaviour of a fractured rock mass consisting of numerous fractures and rocks cannot be predicted by simple applications of the knowledge of individual fractures and/or rocks, due to upscaling complexities involving the hierarchy of scales, heterogeneities, and physical mechanisms as well as the possible emergence of qualitatively different macroscopic properties. In other words, macroscopic phenomena in fractured rocks arise from the many-body effects (i.e. collective behaviour) of numerous interacting fractures and rocks, such that the emergent properties at the fracture system scale are much richer than those of individual components. Hence, *more is different*! This paper gives a discussion on the mechanism of emergence in fractured media from a combined statistical physics and rock mechanics perspective, and further presents a multiscale conceptual framework to link microscopic responses of single fractures/rocks to macroscopic behaviour of rock masses consisting of many fractures and rocks. This framework can serve as a useful tool to bridge experimentally-established constitutive relationships of fracture/rock samples at the laboratory scale to phenomenologically-observed macroscopic properties of fractured rock masses at the site scale.

1. Introduction

Understanding and predicting the macroscopic behaviour of fractured rock masses are of central importance for resolving many core issues in rock engineering applications, such as excavation of deep tunnels, evaluation of slope stability, exploitation of subsurface energy, isolation of radioactive waste, and minimisation of anthropogenic seismicity. Over the past decades, various empirical laws have been proposed to macroscopically describe rock mass properties and responses in situ (Barton et al., 1974; Bieniawski, 1989; Hoek and Brown, 1997; Martin et al., 1999), but the physics behind these empirical criteria is usually elusive, making their domains of validity ambiguous or input parameters difficult to constrain. On the other hand, constitutive relationships of rocks and fractures have been extensively investigated and well established based on experimental studies of core samples at the laboratory scale (Bandis et al., 1983; Cook, 1992; Hoek and Martin, 2014; Jaeger et al., 2007). However, the large-scale behaviour of a fractured rock mass cannot be predicted by simple applications of the knowledge of small-scale individual fracture/rock samples, due to the upscaling complexities involving the hierarchy of scales, heterogeneities, and physical mechanisms as well as the possible emergence of qualitatively different macroscopic phenomena/properties. Great efforts have been devoted to study the effects of scale and heterogeneity on the bulk behaviour of fractured rock masses, but the physical mechanisms that connect different geological scales are still poorly understood. This missing link hinders our fundamental understanding and predictive capability of many complex phenomena in fractured rocks.

This paper aims to present a discussion on the mechanism of emergence in fractured rocks from a combined statistical physics and rock mechanics perspective, and further present a multiscale conceptual framework to link microscopic responses of single fractures/rocks to macroscopic behaviour of rock masses consisting of many fractures and rocks. Here, the term “microscopic” is used to describe the behaviour of individual fractures/rocks on the support scale of their constitutive descriptions, while “macroscopic” refers to the overall properties and responses of fractured rock masses at the system scale. The micro-to-macro link is through “mesoscopic” processes that operate across scales involving multiple fractures and rocks. The rest of the paper is organised as follows. Section 2 discusses the hierarchical nature of fractured rock masses and elaborates the mechanism of emergence in such a complex

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https://doi.org/10.1016/j.rockmb.2023.100080
Received 16 July 2023; Accepted 4 August 2023

system comprising many interacting constituents. Section 3 presents a multiscale framework that bridges microscopic processes to macroscopic phenomena in fractured media honouring the intrinsic hierarchy of the system. Finally, some recommendations for future research are provided in section 4.

2. Fractured rock mass as a complex system

Fractured rock mass comprises a large number of interlocking/interacting rock blocks separated by numerous discontinuity structures that ubiquitously exist over multiple length scales (Bonnet et al., 2001; Lei and Wang, 2016) and actively behave and interact in the subsurface (Rutqvist and Stephansson, 2003; Tsang, 1991). Thus, fractured rock mass is a "complex system" (Ladyman et al., 2013; Siegenfeld and Bar-Yam, 2020) composed of many constituent components (i.e. fractures and rocks) that interact with each other across spatiotemporal scales. Due to the many-body effects (i.e. collective behaviour) of those interacting elements, such a complex system could macroscopically exhibit distinct properties such as emergence, self-organisation, nonlinearity, and feedback loops that cannot be anticipated from individual components alone (Bak, 1996; Sornette, 2006). Diverse, sometimes qualitatively different, macroscopic phenomena could emerge as a result of the collective dynamics of numerous interacting components in the system (Siegenfeld and Bar-Yam, 2020; Sornette, 2002, 2006). Hence, *more is different*, as originally suggested by the theoretical physicist Philip Anderson in the context of condensed matter physics (Anderson, 1972), which is considered to be equally applicable to rock mechanics. A complex system is associated with the twin difficulties of scale and complexity, and at each level of complexity entirely new properties appear, such that the system exhibits properties that its parts do not have on their own, for which reductionism breaks down (Anderson, 1972). Thus, a rock mass as a whole is more than or even very different from the sum of its parts (i.e. fractures and intact rocks), i.e.:

$$\text{Rock mass} \neq \sum \text{Fractures} + \sum \text{Intact rocks}. \tag{1}$$

For many decades, in the rock mechanics community, it has been tacitly assumed that a rock mass equals to fractures plus intact rocks, and the behaviour of a rock mass can be understood by decomposing it into smaller pieces and characterising these pieces completely. However, from the statistical physics point of view, this commonly assumed equation (i.e. rock mass = fractures + intact rocks) is incorrect or at least incomplete. The large-scale behaviour of a fractured rock mass as a complex system cannot be derived by knowing only the detailed properties of individual fractures and/or rocks. It is essential to also know the physical mechanisms that govern the nonlinear interactions among these constituent components across spatiotemporal scales in such a complex system, in order to understand and predict macroscopically emerged collective phenomena in fractured rock masses.

The complexity of fractured media may be governed by three aspects of hierarchy: scales, heterogeneities, and physical mechanisms (Fig. 1). First, fractures exist over almost all relevant length scales, ranging from microcracks (down to the μm-mm scale) to joints (at the cm-hm scale) and faults (up to the 1000 km scale) (Allegre et al., 1982; Bonnet et al., 2001), dissecting the uppermost crust into rock blocks of various shapes and sizes. It is worth pointing out that rock blocks could always contain defects like microcracks inside, but may be practically considered as "intact rocks" with properties defined based on e.g. effective medium approximations (Jaeger et al., 2007) if defects are very small compared to the block size. Many properties pertaining to these multiscale fractures and rocks are scale dependent, such as fracture stiffness and aperture (Bandis et al., 1981; Olson, 2003) as well as intact rock strength and modulus (Hoek and Brown, 1980; Yoshinaka et al., 2008). Due to such scale effects, larger-sized fractures with lower stiffnesses and wider apertures tend to be mechanically more compliant and hydraulically more conductive, while larger-sized rock blocks with lower strengths and moduli may accommodate more deformation and failure. The second aspect of hierarchy is related to the organisation of multiscale structural heterogeneities. For instance, fault zones usually have complicated internal structures consisting of fault core(s) and damage zones, with the latter characterised by a dense network of off-fault secondary fractures (Ben-Zion and Sammis, 2003; Caine et al., 1996; Faulkner et al., 2010); another example is the so-called process zone around the tips of a fracture, where extensive microcracks grow and coalesce (Anders et al., 2014). Such a composite system of multiscale heterogeneities behaves in a hierarchical manner, where a higher-rank structure may exert a strong control on the performance of surrounding lower-rank features which in turn affect the response of the higher-rank structure. Lastly, physical mechanisms also hierarchically operate in fractured media, which can be

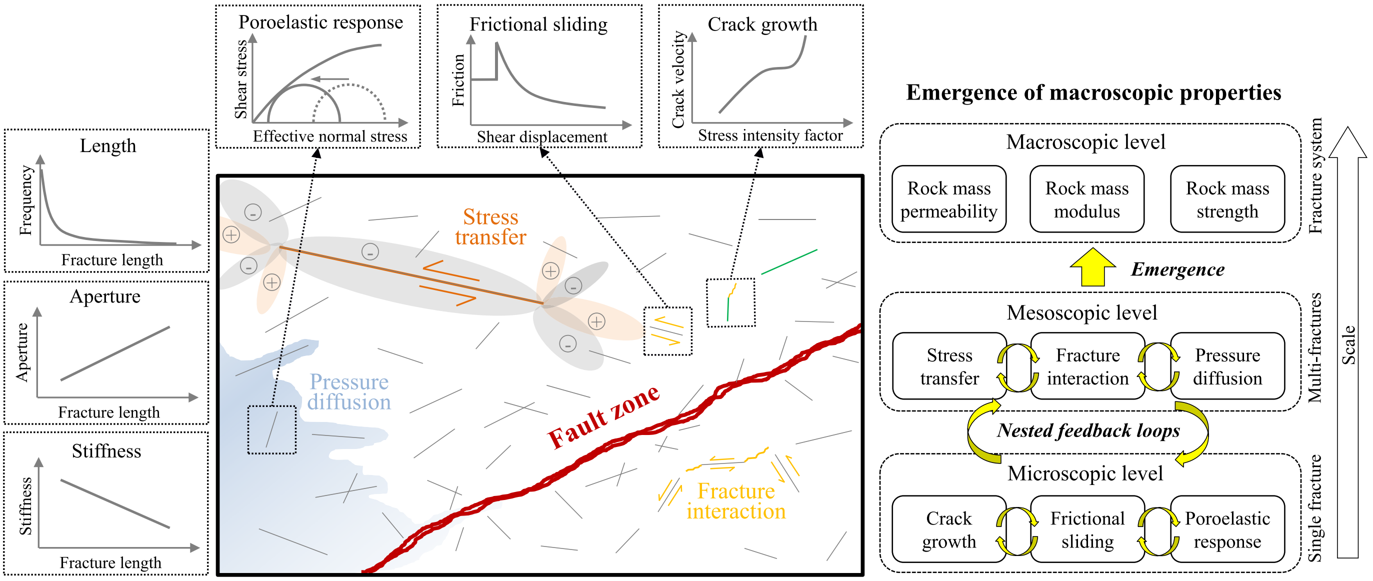

Fig. 1. A schematic illustrating the mechanism of emergence of macroscopic properties in fractured rocks driven by a coupled hierarchy of scales, heterogeneities, and physical mechanisms. Here, the microscopic processes, mesoscopic interactions, and macroscopic properties listed in the right panel are only selected examples for demonstrating the concept.

in general classified into microscopic processes (e.g. frictional sliding, crack growth, poroelastic response, etc. that operate in each individual fracture/rock) and mesoscopic processes (e.g. stress transfer, pressure diffusion, and fracture interaction, etc. that involve multiple fractures/rocks). These microscopic and mesoscopic processes could couple with each other across spatiotemporal scales in fractured media and form nonlinear feedback loops (Fig. 1, right panel). For example, within the microscopic level, the frictional sliding coupled with local pore pressure buildup in a fracture may promote stress concentration in the tip region, which would in turn accelerate the frictional sliding; within the mesoscopic level, stress transfer may drive fracture interaction, which could in turn enhance stress transfer towards the far field. Across the microscopic and mesoscopic levels, stress transfer coupled with poroelasticity can result in overpressure or underpressure at different locations, which would drive pore pressure diffusion across scales and perturbs the stress state over space; fracture interaction may promote, and simultaneously be enhanced by, crack growth in between pre-existing fractures which undergo frictional sliding and poroelastic response. It is the coupled hierarchy of scales, heterogeneities, and physical mechanisms that accommodates a complex interplay of structural heterogeneities and physical processes across scales in fractured media, which drives the emergence and evolution of macroscopic phenomena in the system (Fig. 1).

Fractured rock masses exhibit different levels of complexity at different scales (Fig. 2). For many practical rock engineering problems, we are mainly interested in some of the properties at the large scale, such as the rock mass permeability, modulus, and strength. Characterising all the details, such as the distribution and behaviour of all individual fractures, is very challenging and may not be necessary. Sometimes, a precise description of microscopic details may be unimportant for the prediction of macroscopic properties, where the system might renormalise itself at the large scale with little sensitivity to some of the small-scale details. However, it is clear that predicting the large-scale properties of hierarchical fractured rocks involves more than a simple average of smaller-scale properties. It is essential to consider these hierarchical characteristics and cross-scale interactions in order to understand, interpret, and/or predict the macroscopic behaviour of fractured rock masses.

3. A multiscale conceptual framework for modelling collective phenomena in fractured rocks

Based on the combined statistical physics and rock mechanics perspective, a multiscale conceptual framework is presented for understanding and modelling the emergence of macroscopic phenomena in hierarchical fractured rocks, arising from coupled microscopic and mesoscopic processes and the many-body effects of numerous interacting fractures and rocks.

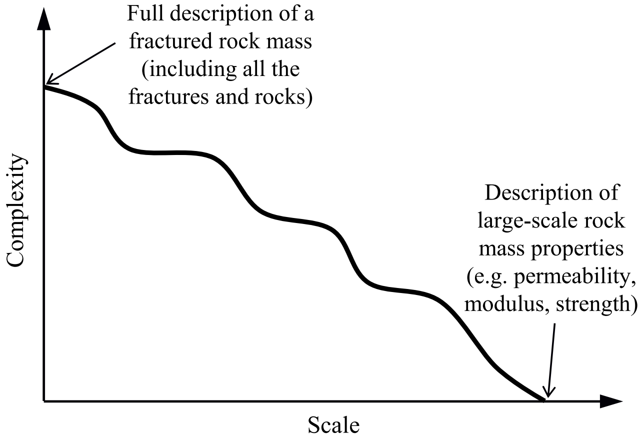

Fig. 2. Variation of complexity as a function of scale in fractured rock masses.

First, it is essential to faithfully honour the discontinuous nature of rock masses by explicitly describing the geometrical distribution and organisation of multiscale fracture populations in rock (Lei et al., 2017). Special attention needs to be paid on realistically representing the topological properties (e.g. connectivity) of fracture networks (Sanderson and Nixon, 2018) due to their important control on processes and phenomena in crustal rocks. No *a priori* assumption should be made about the representative elementary volume when studying hierarchical fractured rocks that may have no characteristic length scale (Bonnet et al., 2001). Relevant material properties (e.g. stiffness, aperture, modulus, strength, etc.) for individual fractures/rocks may be defined according to their geometrical dimensions and prescribed scaling relationships. The hierarchical organisation of multiscale structural heterogeneities, e.g. in fault zones, can be modelled either explicitly with both higher-rank and lower-rank structures fully depicted (Lei and Tsang, 2022) or implicitly with the effects of lower-rank structures considered in the parameterisation of matrix properties nearby higher-rank structures (Cappa and Rutqvist, 2011).

We then need to solve the coupled multiphysical processes operating within and across microscopic and mesoscopic levels (see Fig. 1). The microscopic responses of each individual fracture/rock can be prescribed to follow certain constitutive relationships, which may be either theoretically assumed based on mathematical simplifications (e.g. elastic-perfectly plastic law for fracture shear) or empirically obtained based on laboratory experiments (e.g. Barton-Bandis criterion for fracture shear). These constitutive relationships govern the correlation of local multiphysical state variables (e.g. stress, strain, pore pressure, temperature, etc.) within each individual fracture/rock (Barton et al., 1985; Lei and Barton, 2022). On the other hand, the mesoscopic processes (e.g. stress transfer, pressure diffusion, heat transport, etc.) can be calculated by solving governing equations (e.g. continuity equation, momentum equation, etc.) of the relevant multiphysical fields (Tsang, 1999). This framework therefore simultaneously captures the multiphysical responses that occur microscopically at each constituent and the multiphysical interactions that operate mesoscopically among different constituents in the system. Under this paradigm, diverse macroscopic phenomena can be spontaneously obtained as emergent properties physically arising from coupled microscopic and mesoscopic processes in conjunction with the collective behaviour of a large population of interacting fractures and rocks.

Below, I briefly list a few examples to illustrate how this framework can be applied to study the emergence of complex macroscopic phenomena in fractured rocks (for more details, the reader can refer to the papers cited). By microscopically prescribing Coulomb law and Griffith law to respectively describe the frictional sliding and crack growth, and further mesoscopically computing stress transfer and fracture interaction, the emergence of qualitatively distinct stress dispersion phenomena in fractured media, varying from homogeneous to strongly heterogeneous regimes, is captured (Fig. 3) (Lei and Gao, 2018, 2019). By microscopically representing fracture closure and shear responses based on the experimentally-established Barton-Bandis joint constitutive relationship and further mesoscopically solving stress transfer, pressure diffusion, and fracture interaction (Lei et al., 2016), the emergence of qualitatively different macroscopic flow patterns in fracture media, ranging from disconnected to channelled and distributed regimes, is captured (Fig. 4) (Lei et al., 2015, 2020). By microscopically describing the seismic response of individual fractures based on the displacement discontinuity method and mesoscopically calculating the wave transport and interference within fracture networks, the emergence of diverse macroscopic wavefield behaviours, transitioning from propagation to diffusion and to weak/strong localisation as well as delocalisation regimes, is captured (Fig. 5) (Lei, 2022; Lei and Sornette, 2021a, 2021b). Clearly, all these macroscopic phenomena captured cannot be anticipated from the constitutive relationship prescribed at the microscopic level.

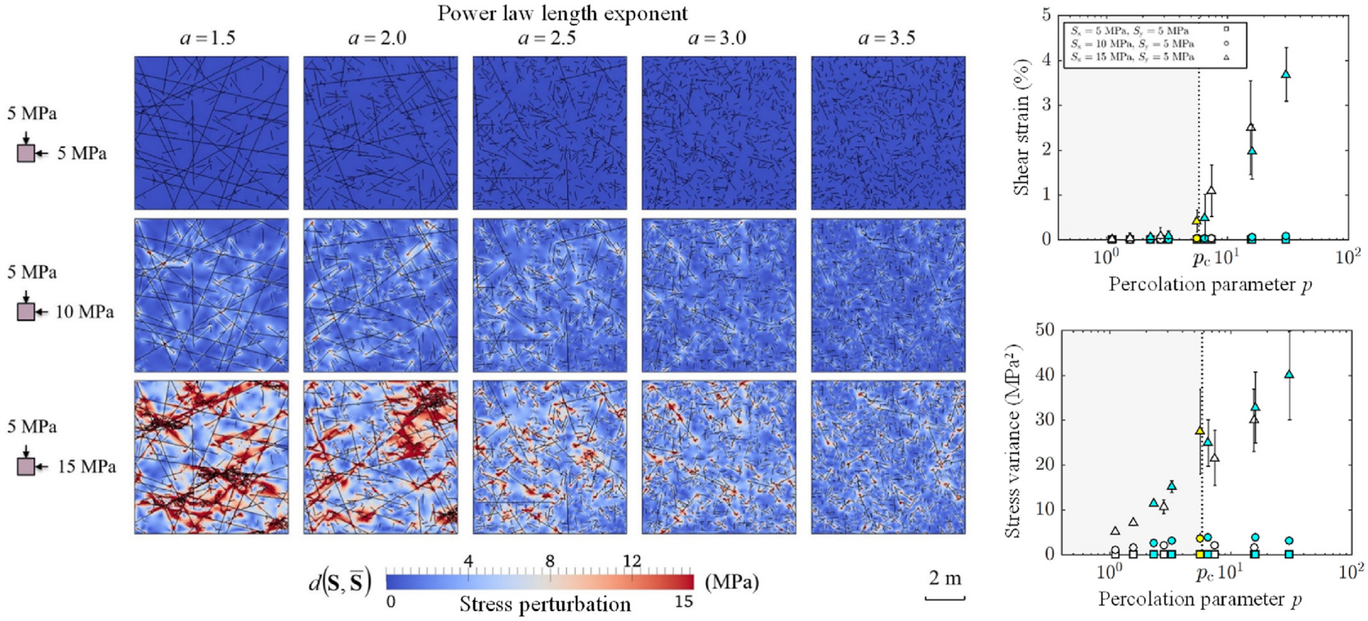

Fig. 3. Emergence of qualitatively different stress dispersion patterns, varying from homogeneous to strongly heterogeneous regimes, in fractured rocks, resulting in connectivity-dependent rock mass deformability and stress variability (Lei and Gao, 2018).

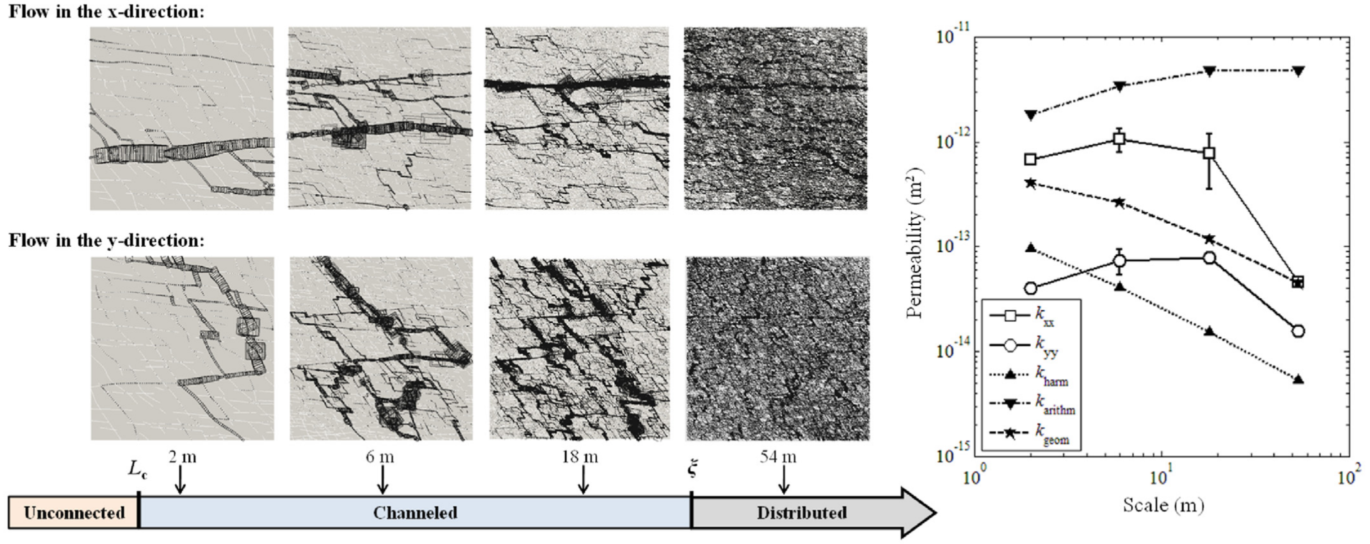

Fig. 4. Emergence of qualitatively different macroscopic flow patterns, ranging from disconnected to channelled and distributed regimes, in fractured rocks as a function of scale, resulting in scale-dependent rock mass permeability variation (Lei et al., 2015).

This framework can be implemented based on different numerical methods (e.g. finite element method, discrete element method, finite difference method, boundary element method, or hybrid methods) (Jing, 2003; Lei et al., 2017). It can be viewed as a generic upscaling tool to explore the consequences of microscopic and mesoscopic processes on macroscopic phenomena in fractured media, and also serve as a multi-angle testing tool to examine macroscopic laws and, in turn, microscopic hypotheses. Furthermore, it can be used as a tool to bridge experimentally-derived constitutive relationships of fracture/rock samples at the laboratory scale to phenomenologically-observed macroscopic properties of fractured rock masses at the site scale. This framework is conceptually different from the conventional fractured media upscaling approaches such as the effective medium theory (Kachanov, 1992) which assumes scale independence beyond a certain scale and the renormalisation group theory (Madden, 1983) which assumes scale invariance across all scales. Instead, it is essential to understand how fractured media perform at different scales, because the system behaviour may vary as a function of scale due to the coupled hierarchy of scales, heterogeneities, and physical mechanisms.

4. Recommendations for future research

This paper gives a discussion, from a combined statistical physics and rock mechanics perspective, on the mechanism of emergence in fractured media as a complex system consisting of numerous interacting constituents. A conceptual framework is further presented for modelling macroscopically emerged collective phenomena in fractured rocks characterised by the presence and interaction of multiscale fractures and

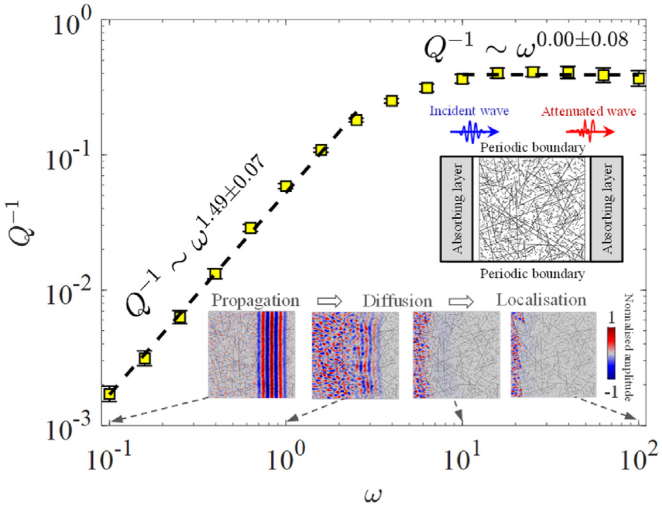

Fig. 5. Emergence of qualitatively different macroscopic wavefield phenomena, transitioning from propagation to diffusion and to localisation, as a function of the dimensionless angular frequency ω, resulting in a two-branch power law frequency-dependency of the inverse quality factor Q^{-1} (Lei and Sornette, 2021a).

rocks. This framework that connects microscopic and mesoscopic processes to macroscopic phenomena across different geological scales may provide an avenue to decipher the hidden cross-scale nonlinear interactive mechanisms in fractured media that remain poorly understood from first principles. Such interactive mechanisms may include interference (multiple entities affect each other), coalescence (multiple entities merge into a larger one), clustering (multiple events spatially/temporally occur closely together), and cascading (multiple events successively occur with each built upon its preceding one), which are sensitive to the organisation and connectivity of entities in the complex system and may invoke nested feedback loops across scales. Further studies are needed to understand how these nonlinear interactions emerge and evolve in fractured media as a result of the complex interplay of structural heterogeneities and physical processes across spatiotemporal scales.

Another recommendation for future research is to use this framework as a tool to bridge constitutive relationships obtained from laboratory-scale experiments to empirical laws proposed for site-scale rock mass properties. Many empirical laws in rock mechanics, e.g. the Hoek-Brown failure criterion (Hoek and Brown, 1997), require to determine some input parameters related to rock mass structures, e.g. geological strength index (Hoek et al., 1995) or rock mass rating (Bieniawski, 1989), in order to estimate site-scale rock mass properties based on laboratory-scale intact rock measurements. However, the determination of these parameters largely relies on empirical knowledge and could be subject to large uncertainties. Furthermore, the current approach for estimating these parameters often finds great difficulty when dealing with complex geological conditions (Aksoy, 2008; Marinos et al., 2005), e.g. strong heterogeneity/anisotropy and high in-situ stress/pore pressure levels that are however inevitably encountered in most rock engineering practice (Barton and Quadros, 2015; Hoek et al., 1995; Hoek and Brown, 1980). This difficulty is due to the fact that the problem of upscaling in fractured media involves not only scales and heterogeneities (which may be somewhat implicitly considered by these empirical parameters), but also physical mechanisms that can possess strong nonlinearity across scales leading to the emergence of qualitatively different macroscopic properties (which are difficult to predict by solely using these empirical parameters). Hence, it is of central importance to further our fundamental understanding of the hierarchy of physical mechanisms and its interplay with structural heterogeneities across scales, by taking advantage of fast-growing computing techniques. It is recommended to use this framework as a testing platform to revisit some of the main empirical laws in rock mechanics, e.g. the Hoek-Brown failure criterion (Hoek and Brown, 1997), the Voight's law of time-to-failure (Voight, 1989), the predictive law for spalling depth (Martin et al., 1999), etc., to better understand their physical origins, clarify their validity ranges, and ultimately constrain their input parameters using physics-based arguments. This framework can also be used to capture the emergence of complex site-scale phenomenological observations and interpret their underlying physical mechanisms, as demonstrated in our recent work on Gotthard Base Tunnel-induced ground deformation (Zhao et al., 2023).

Some related open questions may also deserve further investigations. It is well known that fractures in rock usually exhibit a fractal or power law behaviour without a characteristic length scale (Bonnet et al., 2001), but numerical models can only represent the fracture population within a limited range on length scale due to the resolution constraint. It might be important to know how much microscopic details need to be incorporated in a numerical model in order to adequately capture the emergence of macroscopic properties in fractured media, or equivalently speaking, below what scale microscopic details become unimportant. This can possibly be achieved by systematically diagnosing the consequences of microscopic effects on macroscopic properties of rock masses.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

Q.L. is grateful for the support from the Swiss National Science Foundation (Grant No. 189882) and the National Natural Science Foundation of China (Grant No. 41961134032).

References

Aksoy, C.O., 2008. Review of rock mass rating classification: historical developments, applications, and restrictions. J. Min. Sci. 44, 51–63. https://doi.org/10.1007/s10913-008-0005-2.

Allegre, C.J., Le Mouel, J.L., Provost, A., Allègre, C.J., Le Mouel, J.L., Provost, A., 1982. Scaling rules in rock fracture and possible implications for earthquake prediction. Nature. https://doi.org/10.1038/297047a0.

Anders, M.H., Laubach, S.E., Scholz, C.H., 2014. Microfractures: a review. J. Struct. Geol. 69, 377–394. https://doi.org/10.1016/j.jsg.2014.05.011.

Anderson, P.W., 1972. More is different. Science 177, 393–396. https://doi.org/10.1126/science.177.4047.393.

Bak, P., 1996. How Nature Works: the Science of Self-Organized Criticality. Springer-Verlag, New York.

Bandis, S.C., Lumsden, A.C., Barton, N.R., 1983. Fundamentals of rock joint deformation. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 20, 249–268. https://doi.org/10.1016/0148-9062(83)90595-8.

Bandis, S.C., Lumsden, A.C., Barton, N.R., 1981. Experimental studies of scale effects on the shear behaviour of rock joints. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 18, 1–21. https://doi.org/10.1016/0148-9062(81)90262-X.

Barton, N., Bandis, S., Bakhtar, K., 1985. Strength, deformation and conductivity coupling of rock joints. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 22, 121–140. https://doi.org/10.1016/0148-9062(85)93227-9.

Barton, N., Lien, R., Lunde, J., 1974. Engineering classification of rock masses for the design of tunnel support. Rock Mech. 6, 189–236. https://doi.org/10.1007/BF01239496.

Barton, N., Quadros, E., 2015. Anisotropy is everywhere, to see, to measure, and to model. Rock Mech. Rock Eng. 48, 1323–1339. https://doi.org/10.1007/s00603-014-0632-7.

Ben-Zion, Y., Sammis, C.G., 2003. Characterization of fault zones. Pure Appl. Geophys. 160, 677–715. https://doi.org/10.1007/PL00012554.

Bieniawski, Z.T., 1989. Engineering Rock Mass Classifications: A Complete Manual for Engineers and Geologists in Mining, Civil, and Petroleum Engineering. Wiley, New York.

Bonnet, E., Bour, O., Odling, N.E., Davy, P., Main, I., Cowie, P., Berkowitz, B., 2001. Scaling of fracture systems in geological media. Rev. Geophys. 39, 347–383. https://doi.org/10.1029/1999RG000074.

Caine, J.S., Evans, J.P., Forster, C.B., 1996. Fault zone architecture and permeability structure. Geology 24, 1025–1028. https://doi.org/10.1130/0091-7613(1996)024<1025:FZAAPS>2.3.CO;2.

Cappa, F., Rutqvist, J., 2011. Modeling of coupled deformation and permeability evolution during fault reactivation induced by deep underground injection of CO2. Int. J. Greenh. Gas Control 5, 336–346. https://doi.org/10.1016/j.ijggc.2010.08.005.

Cook, N.G.W., 1992. Natural joints in rock: mechanical, hydraulic and seismic behaviour and properties under normal stress. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 29, 198–223. https://doi.org/10.1016/0148-9062(92)93656-5.

Faulkner, D.R., Jackson, C.A.L., Lunn, R.J., Schlische, R.W., Shipton, Z.K., Wibberley, C.A.J., Withjack, M.O., 2010. A review of recent developments concerning the structure, mechanics and fluid flow properties of fault zones. J. Struct. Geol. 32, 1557–1575. https://doi.org/10.1016/j.jsg.2010.06.009.

Hoek, E., Brown, E., 1980. Underground Excavations in Rock. Instu Min Metall, London.

Hoek, E., Brown, E.T., 1997. Practical estimates of rock mass strength. Int. J. Rock Mech. Min. Sci. 34, 1165–1186. https://doi.org/10.1016/S1365-1609(97)80069-X.

Hoek, E., Kaiser, P.K., Bawden, W.F., 1995. Support of Underground Excavation in Hard Rock. Balkema, Rotterdam.

Hoek, E., Martin, C.D., 2014. Fracture initiation and propagation in intact rock – a review. J. Rock Mech. Geotech. Eng. 6, 287–300. https://doi.org/10.1016/j.jrmge.2014.06.001.

Jaeger, J.C., Cook, N.G.W., Zimmerman, R.W., 2007. Fundamentals of Rock Mechanics, fourth ed. Blackwell Publishing, Oxford.

Jing, L., 2003. A review of techniques, advances and outstanding issues in numerical modelling for rock mechanics and rock engineering. Int. J. Rock Mech. Min. Sci. 40, 283–353. https://doi.org/10.1016/S1365-1609(03)00013-3.

Kachanov, M.L., 1992. Effective elastic properties of cracked solids: critical review of some basic concepts. Appl. Mech. Rev. https://doi.org/10.1115/1.3119761.

Ladyman, J., Lambert, J., Wiesner, K., 2013. What is a complex system? Eur. J. Phil. Sci. https://doi.org/10.1007/s13194-012-0056-8.

Lei, Q., 2022. Impact of fracture normal and shear stiffnesses on the scattering attenuation of P and S waves in a naturally fractured rock. J. Appl. Geophys. 206, 104784. https://doi.org/10.1016/j.jappgeo.2022.104784.

Lei, Q., Barton, N., 2022. On the selection of joint constitutive models for geomechanics simulation of fractured rocks. Comput. Geotech. 145, 104707. https://doi.org/10.1016/j.compgeo.2022.104707.

Lei, Q., Gao, K., 2019. A numerical study of stress variability in heterogeneous fractured rocks. Int. J. Rock Mech. Min. Sci. 113, 121–133. https://doi.org/10.1016/j.ijrmms.2018.12.001.

Lei, Q., Gao, K., 2018. Correlation between fracture network properties and stress variability in geological media. Geophys. Res. Lett. 45, 3994–4006. https://doi.org/10.1002/2018GL077548.

Lei, Q., Latham, J.-P., Tsang, C.-F., 2017. The use of discrete fracture networks for modelling coupled geomechanical and hydrological behaviour of fractured rocks. Comput. Geotech. 85, 151–176. https://doi.org/10.1016/j.compgeo.2016.12.024.

Lei, Q., Latham, J.-P., Tsang, C.-F., Xiang, J., Lang, P., 2015. A new approach to upscaling fracture network models while preserving geostatistical and geomechanical characteristics. J. Geophys. Res. Solid Earth 120, 4784–4807. https://doi.org/10.1002/2014JB011736.

Lei, Q., Latham, J.-P., Xiang, J., 2016. Implementation of an empirical joint constitutive model into finite-discrete element analysis of the geomechanical behaviour of fractured rocks. Rock Mech. Rock Eng. 49, 4799–4816. https://doi.org/10.1007/s00603-016-1064-3.

Lei, Q., Sornette, D., 2021a. Transport and localization of elastic waves in two-dimensional fractured media: consequences on scattering attenuation. J. Geophys. Res. Solid Earth 126, e2020JB021178. https://doi.org/10.1029/2020JB021178.

Lei, Q., Sornette, D., 2021b. Anderson localisation and reentrant delocalisation of tensorial elastic waves in two-dimensional fractured media. Europhys. Lett. 136, 39001. https://doi.org/10.1209/0295-5075/ac225d.

Lei, Q., Tsang, C.-F., 2022. Numerical study of fluid injection-induced deformation and seismicity in a mature fault zone with a low-permeability fault core bounded by a densely fractured damage zone. Geomech. Energy Environ., 100277 https://doi.org/10.1016/j.gete.2021.100277.

Lei, Q., Wang, X., 2016. Tectonic interpretation of the connectivity of a multiscale fracture system in limestone. Geophys. Res. Lett. 43, 1551–1558. https://doi.org/10.1002/2015GL067277.

Lei, Q., Wang, X., Min, K.-B., Rutqvist, J., 2020. Interactive roles of geometrical distribution and geomechanical deformation of fracture networks in fluid flow through fractured geological media. J. Rock Mech. Geotech. Eng. 12, 780–792. https://doi.org/10.1016/j.jrmge.2019.12.014.

Madden, T.R., 1983. Microcrack connectivity in rocks: a renormalization group approach to the critical phenomena of conduction and failure in crystalline rocks. J. Geophys. Res. 88, 585. https://doi.org/10.1029/JB088iB01p00585.

Marinos, V., Marinos, P., Hoek, E., 2005. The geological strength index: applications and limitations. Bull. Eng. Geol. Environ. 64, 55–65. https://doi.org/10.1007/s10064-004-0270-5.

Martin, C.D., Kaiser, P.K., McCreath, D.R., 1999. Hoek-Brown parameters for predicting the depth of brittle failure around tunnels. Can. Geotech. J. 36, 136–151. https://doi.org/10.1139/t98-072.

Olson, J.E., 2003. Sublinear scaling of fracture aperture versus length: an exception or the rule? J. Geophys. Res. 108, 2413. https://doi.org/10.1029/2001JB000419.

Rutqvist, J., Stephansson, O., 2003. The role of hydromechanical coupling in fractured rock engineering. Hydrogeol. J. 11, 7–40. https://doi.org/10.1007/s10040-002-0241-5.

Sanderson, D.J., Nixon, C.W., 2018. Topology, connectivity and percolation in fracture networks. J. Struct. Geol. 115, 167–177. https://doi.org/10.1016/j.jsg.2018.07.011.

Siegenfeld, A.F., Bar-Yam, Y., 2020. An introduction to complex systems science and its applications. Complexity 2020, 6105872. https://doi.org/10.1155/2020/6105872.

Sornette, D., 2006. Critical Phenomena in Natural Sciences - Chaos, Fractals, Selforganization and Disorder: Concepts and Tools, second ed. Springer, Berlin.

Sornette, D., 2002. Predictability of catastrophic events: material rupture, earthquakes, turbulence, financial crashes, and human birth. Proc. Natl. Acad. Sci. U.S.A. 99, 2522–2529. https://doi.org/10.1073/pnas.022581999.

Tsang, C.-F., 1999. Linking thermal, hydrological, and mechanical processes in fractured rocks. Annu. Rev. Earth Planet Sci. 27, 359–384. https://doi.org/10.1146/annurev.earth.27.1.359.

Tsang, C.-F., 1991. Coupled hydromechanical-thermochemical processes in rock fractures. Rev. Geophys. 29, 537. https://doi.org/10.1029/91RG01832.

Voight, B., 1989. A relation to describe rate-dependent material failure. Science 243, 200–203. https://doi.org/10.1126/science.243.4888.200.

Yoshinaka, R., Osada, M., Park, H., Sasaki, T., Sasaki, K., 2008. Practical determination of mechanical design parameters of intact rock considering scale effect. Eng. Geol. 96, 173–186. https://doi.org/10.1016/j.enggeo.2007.10.008.

Zhao, C., Lei, Q., Zhang, Z., Loew, S., 2023. Causal mechanism of Gotthard Base Tunnel-induced ground deformation: insights from 3D fully-coupled hydro-mechanical simulation and comparison to field measurements. Int. J. Rock Mech. Min. Sci. 170, 105515. https://doi.org/10.1016/j.ijrmms.2023.105515.